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SHORT COMMUNICATION
PREDICTION OF ELASTIC SETTLEMENT OF RECTANGULAR RAFT FOUNDATION — A COUPLED FE–BE APPROACH

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SUMMARY

A numerical method of analysis is proposed for computation of the elastic settlement of raft foundations using a FEM–BEM coupling technique. The structural model adopted for the raft is based on an isoparametric plate bending finite element and the raft–soil interface is idealized by boundary elements. Mindlin's half-space solution is used as a fundamental solution to find the soil flexibility matrix and consequently the soil stiffness matrix. Transformation of boundary element matrices are carried out to make it compatible for coupling with plate stiffness matrix obtained from the finite element method. This method is very efficient and attractive in the sense that it can be used for rafts of any geometry in terms of thickness as well as shape and loading. Depth of embedment of the raft can also be considered in the analysis.

Key words: raft foundation; elastic settlement; semi infinite half-space; layered media

INTRODUCTION

The problem of analysis and design of raft foundations has attracted the attention of engineers and researchers for a long time. This is because the rafts are frequently associated with major multistoried structures founded on different types of soils for controlling settlement to provide safe foundation systems. Since settlement control is one of the major objectives in providing raft foundations, it is of primary importance to predict the settlement for a particular raft–soil system. To predict the settlement of a raft foundation is a complex problem in which a great deal of judgement must be exercised. Settlement of a structure usually consists of

(i) Elastic settlement.
(ii) Consolidation settlement.

In the present paper, a numerical method of analysis is proposed to predict the elastic settlement of rectangular raft foundations.

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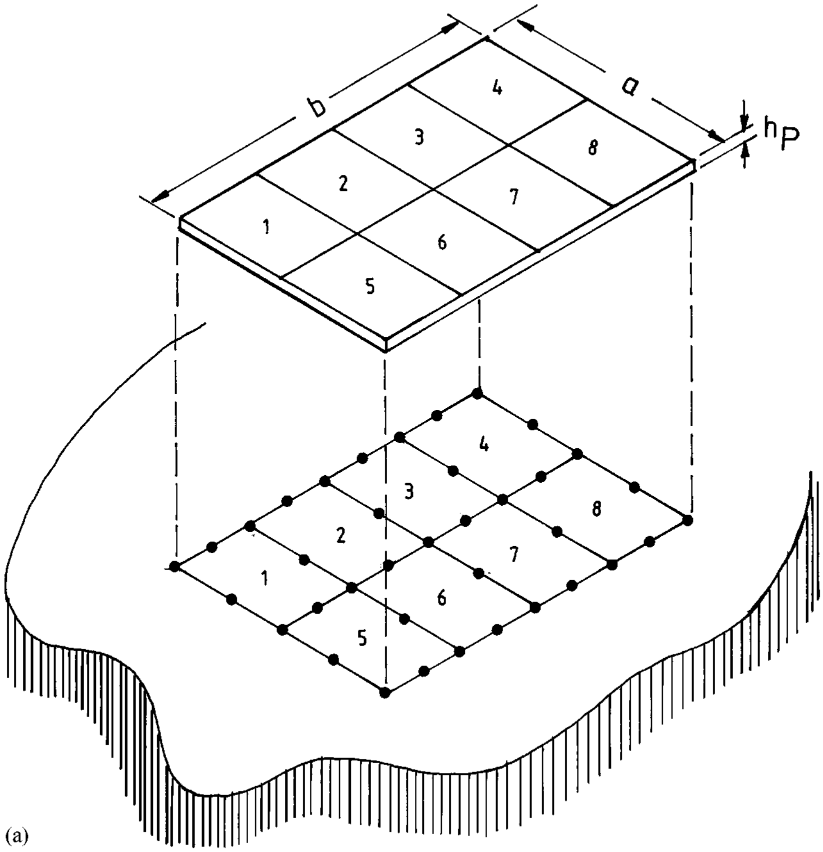
BRIEF REVIEW AND SCOPE

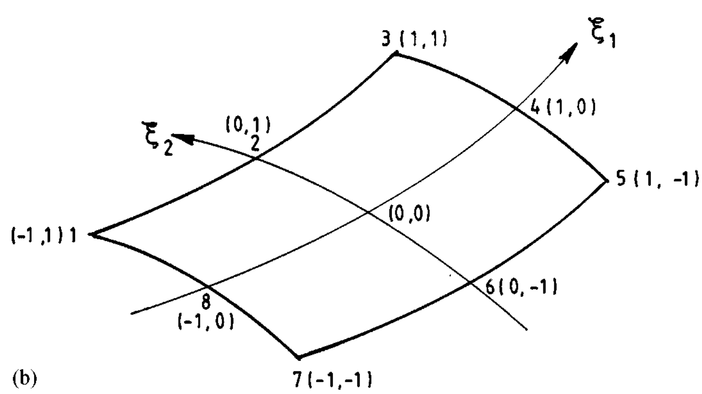
For settlement computations of rafts, frequent use has been made of the one-dimensional method with parameters obtained from oedometer tests (consolidation settlements). Elastic settlement computations are mainly carried out in terms of Young's modulus obtained from different direct and indirect tests. Among the methods used, the solution by Fox[1] has been particularly popular for embeded flexible rectangular foundations. For analysing raft foundations, the raft is treated as a plate on an elastic foundation. The first satisfactory solution for a square raft on elastic foundation of arbitrary flexibility was obtained by Cheung and Zienkiewicz.[2] Cheung and Nag[3] used a similar technique to allow for horizontal contact pressures beneath the raft. The distribution of soil reaction is approximated as concentrated forces at the nodes of the rectangular plate bending elements used for structural modelling of the raft. Svec and Gladwell[4] improved the method of Cheung and Zienkiewicz[2] by assuming a continuous pressure distribution. Fraser and Wardle[5] used the finite element technique for multilayered soil systems. The analysis was restricted to the study of the behaviour of a raft subjected to uniformly distributed loads. Rajapakse and Selvadurai[6] provided a brief literature review on the application of the finite element technique in interaction analysis of plates on elastic foundations.

The boundary element method (BE) was used in the analysis of plates resting on elastic foundations using Winkler foundation model by Katsikadelis and Armenkas,[7, 8] Costa and Brebia[9,10] and two parameter models by Katsikadelis and Kalivokas.[11] Sapountzakis and Katsikadelis[12] applied the boundary element method to unilaterally supported plates on a Winkler foundation. Jianguo *et al.*[13] used a direct boundary integral equation formulation for a Reissner-type thick plate on a Winkler foundation. But BE methods are not very popular for computation of settlement for raft foundations mainly due to parametric limitations. Subgrade modulus obtained directly by a plate load test or by indirect methods can only give soil properties in the vicinity of the tests and is not representative of the whole soil mass.

Kay and Cavagnaro[14] provided a simple and approximate method for settlement computation by using field parameters. Gazetas *et al.*[15] developed a realistic analytical expression for estimating vertical elastic settlement of arbitrarily shaped foundations. Bowles[16] proposed a method for computing the elastic settlement of a foundation on sand deposits using the conventional method by adjusting the influence factor as computed by Steinbrenner.[17] The settlement of a raft foundation depends not only on the soil below, but also on the relative stiffness of the raft and the soil. In the present study, a hybrid approach (FEM-BEM) is employed to predict the settlement. The raft (idealized as a plate) and the soil (elastic foundation) are two separate bodies in unilateral and frictionless contact at their interface. The plate is discretized into isoparametric plate bending finite elements having eight nodes, each node having three degrees of freedom, namely two orthogonal rotations and a vertical displacement. The soil is treated as boundary elements having identical discretisation so that a node to node correspondence is maintained at the interface [Figures 1(a) and (b)]. Vertical displacement compatibility is maintained at the interface. Any complex structural form and loading can be analysed by using this method by maintaining compatible degrees of freedom at the interfacial nodes. This method is also suitable for analysing rafts which are placed at some depth below the ground level (which is generally done in practice).



(a)



(b)

Figure 1 (a) Plate on elastic continuum—discretization of soil foundation system in plate finite elements and soil boundary elements; (b) Two-dimensional isoparametric quadrilateral quadratic element

MATHEMATICAL FORMULATION

Because of the inherent singularities in the expressions from Mindlin's solution for the displacement it is not possible to constitute the stiffness matrix in its usual sense. On the other hand, the displacement field due to a distributed load obtained by integration of these expressions over a finite area is regular everywhere. Hence, it is possible to express the nodal displacements in terms of nodal stress intensities. This relation, may be evaluated by directly integrating the Mindlin's singular expressions[18] for a concentrated load. The Cartesian displacement vector at a point P due to a traction distributed over an infinitesimal area $\mathrm{d}s(Q)$ at Q may be written as

$$\mathrm{d}u_i(P) = U_{ij}(P, Q)t_j(Q)\,\mathrm{d}s(Q) \tag{1}$$

where $U_{ij}(P, Q)$ represent the displacement at P in the ith direction due to traction $t_j(Q)$ in the jth direction at Q. Integrating over the entire soil–structure interface the expressions for displacement at P may be written as

$$u_i(P) = \int_s U_{ij}(P\ Q)t_j(Q)\,\mathrm{d}s(Q) \tag{2}$$

The traction vector at an arbitrary point Q is related to the corresponding nodal components by the interpolation

$$t_j(Q) = N_c(t_j)_c \tag{3}$$

where N_c is the shape function and $(t_j)_c$ is the corresponding traction at the nodal point. So equation (2) can now be written as

$$u_i(P) = \int_s U_{ij}(P, Q)N_c(t_j)_c\,\mathrm{d}s(Q) \tag{4}$$

In the present formulation the boundary (surface) of the solution domain is divided into a number of interconnected two-dimensional isoparametric quadrilateral quadratic elements. In the intrinsic coordinate system the equation takes the form

$$u_i(P) = \sum_{m=1}^{M} \sum_{c=1}^{8} (t_j)_c \int_{-1}^{+1} \int_{-1}^{+1} U_{ij}(P, Q)N_c(\xi_1, \xi_2)J(\xi_1, \xi_2)\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2 \tag{5}$$

where M is the total number of elements. Integrating the above expression for each node one by one and after subsequent assembly in the global array the displacement vector may be put in the form

$$\{(u_i)_c\} = [A]\{(t_i)_c\} \tag{6}$$

where $\{(u_i)_c\}$ and $\{(t_i)_c\}$ are nodal displacement and stress intensity vectors and $[A]$ is the soil flexibility matrix. Since displacement vanishes as $r \to \infty$ the rigid-body mode is naturally excluded from the solution. Hence, the relation between the distributed nodal stress parameter $(t_i)_c$ and displacement $(u_i)_c$ is unique and non-singular. Thus the inverse relation can be expressed as

$$\{(t_i)_c\} = [A]^{-1}\{(u_i)_c\} \tag{7}$$

Coupling of FE and BE matrices

A problem arises in coupling BE & FE matrices due to the use of 'traction' and 'force', respectively, as variables. Nodal tractions can be transformed into equivalent nodal forces by simple energy considerations and shape functions and the relationship is given by

$$\{(F_i)_c\} = [M]\{(t_i)_c\} \tag{8}$$

where $\{(F_i)_c\}$ is the nodal force vectors due to the soil reaction and $[M]$ is the transformation matrix. Now premultiplying equation (7) by $[M]$

$$[M]\{(t_i)_c\} = [M][A^{-1}]\{u_i)_c\}$$

or

$$\{(F_i)_c\} = [MA^{-1}]\{(u_i)_c\} \tag{9}$$

But, neither the nodal force vectors nor the displacements are known till the nodal force vectors are related to the externally applied loads. These relationships are derived from equations of equilibrium as described below.

The foundation (raft) is idealized as thick plate. Isoparametric plate bending elements are used in the present formulation. They can be used for analysis of both thin and thick plates. The same discretization is used as for the soil boundary element so that a node to node correspondence is obtained. As explained earlier the soil pressure is represented by nodal force vectors at the nodes. So from the kinetics of the soil–plate system

$$K_{\mathrm{p}}\{(u_i)_c\} = -\{(F_i)_c\} + P_0 \tag{10}$$

where K_{p} is the assembled stiffness matrix of the plate and $(u_i)_{\mathrm{c}}$ is the generalized displacement at the nodes and P_0 is the applied external load at the nodes. Substituting the value of $(F_i)_c$ from equation (9)

$$K_{\mathrm{p}}\{(u_i)_c\} = -[MA^{-1}]\{(u_i)_c\} + P_0$$

or

$$[K_{\mathrm{p}} + MA^{-1}]\{(u_i)_c\} = P_0$$

i.e.

$$\{(u_i)_c\} = [K_{\mathrm{p}} + MA^{-1}]^{-1}P_0 \tag{11}$$

and

$$\{(u_i)_c\} = [K_{\mathrm{ps}}]^{-1}P_0$$

where

$$[K_{\mathrm{ps}}] = [K_{\mathrm{p}} + MA^{-1}]$$

K_{ps} is the combined stiffness matrix. By solving the above equation displacement parameters can be obtained.

NUMERICAL EXAMPLES

On the basis of the mathematical formulation and numerical procedures presented in the previous sections, a computer program has been developed and representative examples have been studied to demonstrate the range of applications of the proposed method.

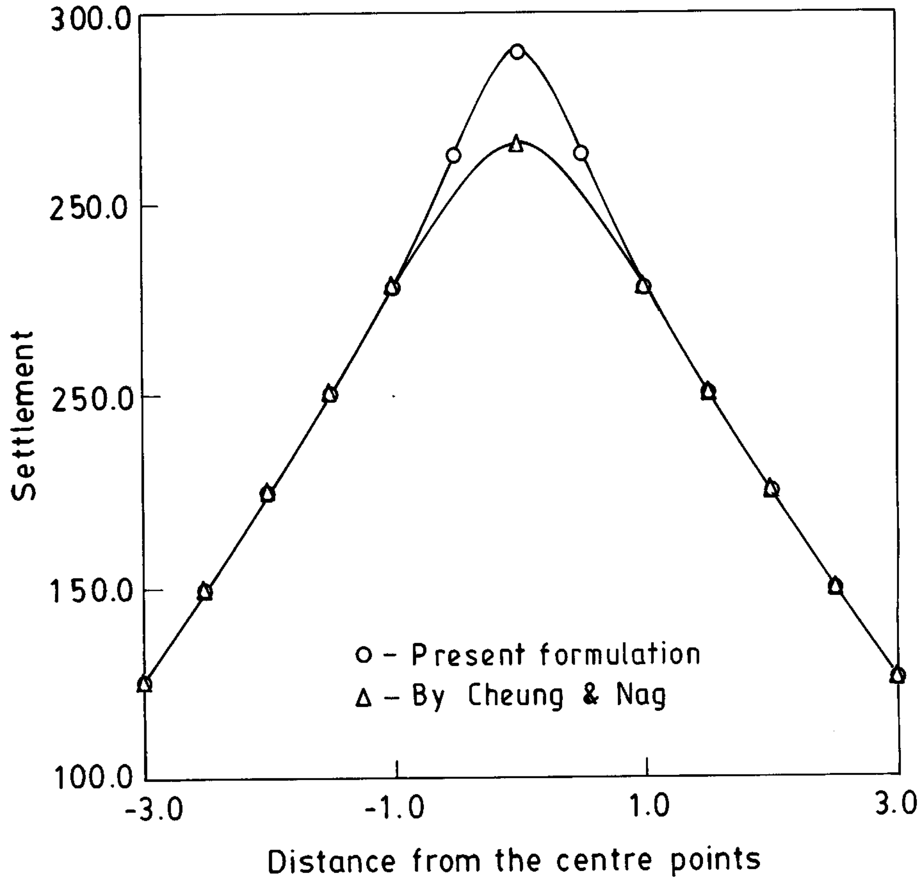


Figure 2 Settlement along the centreline of the centrally loaded square raft on a semi-infinite half-space

Raft on isotropic homogeneous half-space

(a) A square raft of size 6 by 6 and having thickness of 0·8 is subjected to a concentrated load of 1000·0 at the centre. The raft's Young's modulus $E_r = 40{\cdot}0$ and Poission's ratio $\mu_r = 0{\cdot}15$, and for the soil the values are $E_s = 1{\cdot}0$ and $\mu_s = 0{\cdot}15$.[3]

The results obtained are compared graphically in Figure 2. The central settlement for the present study does not match with that given by Cheung and Nag. But the convergence test shows that it converges at this value, the lower value of Cheung and Nag's analysis may be attributed to the coarser mesh and idealization of the raft by four-noded rectangular elements whereas in the present analysis isoparametric plate bending elements are used.

(b) *Effect of depth of embedment on settlement*: To study the effect of depth of embedment on settlement, a comparative analysis is performed for different Poisson's ratios of the soil. The ratio of settlement when the raft is placed at some depth (W_d) to surface settlements ($W_{surface}$) against different depth (d) to width (a) ratio for a square raft for a uniformly

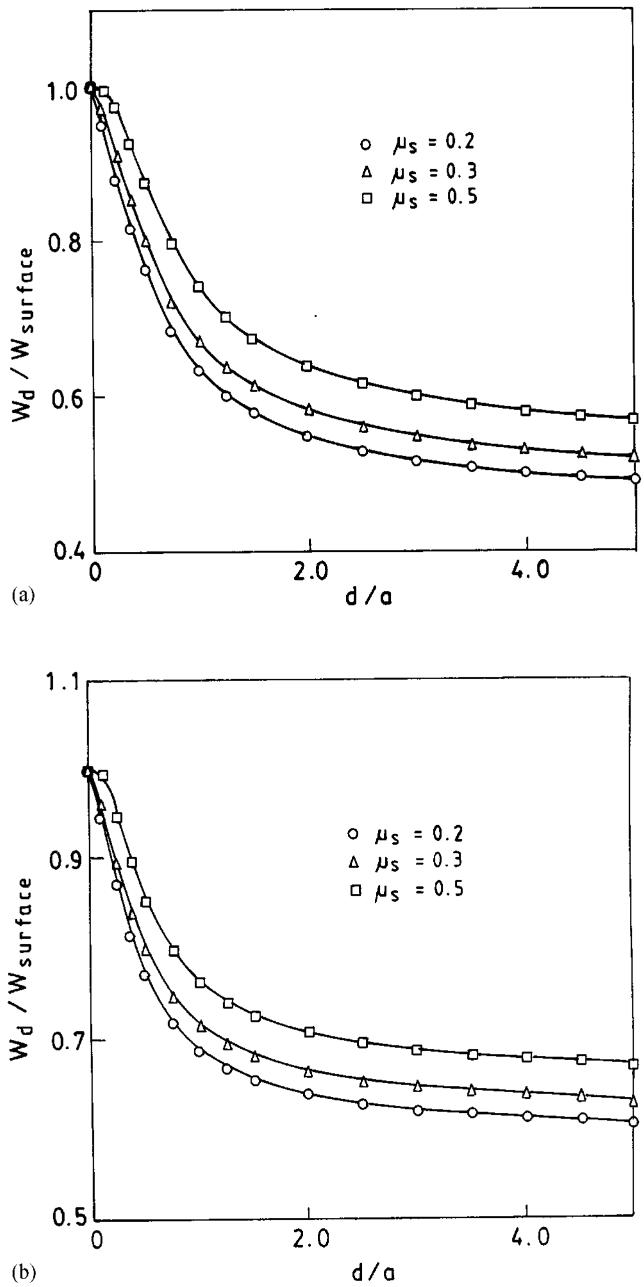


Figure 3 Effect of embedment on settlements of a (a) uniformly loaded square raft in a semi-infinite half-space; (b) a centrally loaded square raft in a semi-infinite half-space

Table I Comparison of predicted and measured settlements. Saving Bank Building[14]

Methods	Center settlements (mm)	Differential settlements (mm)
Conventional: 1D approach	22	16
Finite layer	23	16
Hooke's Law approach	26	—
Kay and Cavangaro	26	12
Proposed method	28	8
Measured settlements	16–18	7–11

distributed or for a central load at centre is plotted in Figure 3(a) and 3(b), respectively. The influence of depth of embedment of the raft can be handled easily since the fundamental solution used for the analysis is Mindlin's half-space solution, by which the soil flexibility matrix of the raft–soil interface can be evaluated at any depth.

Raft on layered media

For all practical purposes the soil mass below a depth of four to five times the width of the foundation has little influence on the foundation settlements. So to compute the settlement of a raft foundation on a layered medium, the depth of soil up to five times the width has been considered. To use the present formulation for a layered media equivalent elastic parameters are estimated by the same technique as followed by Fraser and Wardle.[5]

Example: Raft size: $33{\cdot}5$ m $\times$ $39{\cdot}5$ m, Thickness $= 0{\cdot}9$ m, Loading: uniformly distributed load of 134 kPa. The soil: 2 m from the raft bottom $E_s = 44$ MPa, for next 8 m, $E_s = 60$ MPa followed by a soil having $E_s = 500$ MPa. The value of Poisson's ratio $\mu_s = 0{\cdot}2$ for all soil layers. The value of $E_r = 0{\cdot}25 \times 10^8$ kPa, and Poisson's ratio $\mu_r = 0{\cdot}15$.[14] The comparison of results with other published data and the measured settlements is given in the Table I.

Raft on layered media underlain by a rigid base[5]

The same procedure is followed (as in case of layered media) to estimate the equivalent elastic parameters of the soil system. i.e. the soil is considered to reach up to a depth of five times the width of the raft. At the depth at which the rigid base starts, the value of elastic modulus of the soil is assigned a very high value.

Example: Raft: size: $10{\cdot}0 \times 10{\cdot}0$ m, thickness $= 0{\cdot}5$ m, $E_r = 15{,}000$ MPa, $\mu_r = 0{\cdot}2$, Loading: uniformly distributed load of 100 kPa. The soil: four layers of thickness $10{\cdot}0$ m having moduli of $E_s = 100, 80, 60, 100$ MPa, respectively, followed by a rigid base. The value of Poisson's ratio $\mu_s = 0.3$ for all layers. Comparison of results with published data is shown in Table II.

Rigid raft embedded in halfspace

For low values of foundation-raft stiffness parameter ($\leqslant 3{\cdot}016$) as defined by Gorbunov–Posadov and Serebrjanyi[19] the raft behaves like rigid one. So settlement can be found

Table II Comparison of estimated settlements (raft on layered media underlain by a rigid base)[5]

Methods	Central settlements (m)	Differential settlements (m)
Fraser and Wardle (approximate method)	0·0107	0·0029
Fraser and Wardle (exact results obtained from finite element method)	0·0114	0·0027
Proposed method	0·0106	0·0028

for such a low value of stiffness parameter. The other way is by use of iterative procedures untill nearly uniform settlement is obtained. This is done by altering the thickness untill the differential settlement is less than a specified value (0·1 mm). In the present study the latter technique is followed. Since Mindlin's solution is used, the soil flexibility matrix at the interface can be found out at the required depth of embedment.

Example: Raft: size: 10·5 m $\times$ 27·5 m, $E_r = 15{,}000$ Mpa, $\mu_r = 0{\cdot}2$. Loading: Concentrated load of 8 MN at centre. The soil: Homogeneous elastic half-space having a value of $E_s = 6$ MPa. The value of Poisson's ratio $\mu_s = 0{\cdot}35$. Depth of embedment = 7·5 m.[15] Settlements of the above soil foundation system as estimated by G. Gazetas *et al.* is 46 mm, whereas the predicted settlement by the proposed method is 48 mm.

Rigid plate displacement

For comparing the rigid displacements of a centrally loaded square plate obtained by the proposed method with other published results, the value of the numerical factor (α) as given below is estimated.

$$\alpha = \frac{2aE_s w_r}{(1-\mu_s^2)P}$$

where P is the load applied and w_r is the rigid plate displacement. The value of 'α' obtained by the authors is 0·879 where as the value obtained by Rajapakse and Selvadurai[6] and Caifeng Hu and Hartley[20] are 0·869 and 0·8961, respectively. The corresponding value given by Gorbunov–Posadov and Serebrjanyi[19] is 0·913.

ADVANTAGES OF THE PRESENT METHOD

In the present numerical method the soil is idealized as a semi-infinite elastic continuum. The other alternative method which is followed in the case of a truely three-dimensional analysis is to represent the soil continua as a set of interconnected springs connected to the structure at the interface nodes (Winkler model). This method may be computationally elegant but is unsatisfactory in the sense that the physical nature of the soil is not properly modelled and the spring constant (subgrade modulus) depends not only on the displacement characteristics of the soil but also on the geometry of the given soil–structure system.

Compared to the method used by Cheung and Nag[3] (who also used elastic continuum model for the soil), the proposed method is more versatile in the sense that it can consider the depth of embedment of the raft and also complete raft–soil interface compatibility is ensured. In their analysis a constant pressure is assumed to act on a rectangle (depending on the mesh size) around each nodal point. Compatibility is satisfied at the nodal points only. The present formulation has more flexibility for adaption to other raft shapes since isoparametric elements are used.

CONCLUSIONS

The proposed numerical method for predicting settlement of rectangular raft foundations has a wide range of applications and is suitable for the raft–soil systems generally encountered in practice. It is devoid of the formidable computational problems encountered in finite element analysis due to the infinite lateral extent of the soil continua.

The settlements predicted by the proposed method agree quite well with the published results for different soil–raft systems (as shown graphically and in tabular form for different numerical examples considered). The predicted settlements for the layered media by using equivalent elastic parameters gives very close proximity to the values obtained by finite element analysis. With slight modification the present formulation can also be used to estimate settlement for rafts of any arbitrary shape and stiffness subjected to any type of loading placed at any depth.

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